

# Coupling Circuit Resonators Among Themselves and To Nitrogen-Vacancy Centers in Diamond

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We propose a scheme to couple NV centers in diamond through coplanar waveguide resonators. The central conductor of the resonator is split into several pieces which are coupled strongly with each other via simple capacitive junctions or superconducting Josephson junctions. The NV centers are then put at the junctions. The discontinuity at the junctions induces a large local magnetic field, with which the NV centers are strongly coupled to the circuit resonator. The coupling strength  $g$  between the resonator and the NV center is of order of  $g/2\pi \sim 1\text{--}30$  MHz.

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A nitrogen-vacancy (NV) center in diamond consists of a nitrogen atom substituting a carbon atom and a vacancy trapped adjacent to the substitutional nitrogen. In its ground state, the negatively charged NV center has a spin triplet, which is separated by optical transitions from the excited states. The long spin coherence time and fairly easy optical initialization and read out of the ground spin state make the NV center an excellent candidate for quantum information processor and quantum information storage [1]. The coherent manipulation of the single spin or multiple spins of electrons and nuclei within a single NV center [2–4] or locally interacting NV centers [5] have been demonstrated experimentally. To build a scalable quantum information processors (QIP), however, controlled coupling between distant NV centers is yet to be achieved.

A superconducting coplanar waveguide resonator, defined by a single centimeters-long central conductor between two ground half-planes, has been successfully used as a quantum bus for charge-based superconducting qubits [6, 7] by exploiting a strong *electric* dipole coupling [8, 9]. Building a hybrid quantum device using the superconducting resonator as a quantum bus for the spin qubits (including NV centers) is desirable for the scalable QIP, because then we can take advantage of the scalability and low dissipation of the circuit-QED system as well as the long coherence time of spin qubits. However, it has been limited only to collective excitation of the spin ensemble due to a small *magnetic* dipole coupling of a single spin qubit with the resonator of an order of 10 Hz [10–14]. Using a flux qubit as a mediator between the NV center and the resonator is suggested to enhance the coupling strength recently, but then the short coherence time of the flux qubit limits the coherence time of the entire system [15].

In this paper, we describe how to realize a quantum bus for distant NV centers using a series of superconduct-

ing resonators coupled by either Josephson or capacitive junctions. We note that inserting a Josephson junction into the central conductor of the resonator enhances *local* magnetic field by a factor of  $10^4$  [16, 17]. Interestingly, a simple capacitive junction can also enhance the local magnetic field by the same factor as demonstrated below, which would allow us to circumvent the difficulties of fabricating many Josephson junctions in the central conductor. By putting single NV center at each junction, the magnetic dipole coupling strength between the resonator and the NV centers reaches  $g/2\pi \sim 1\text{--}30$  MHz. The interaction between NV centers can be achieved by exchanging virtual photons and can be turned on and off by bringing them in and out of resonance with the resonator frequency by the external magnetic field. Thus quantum gate operations can be performed in the same manner as the circuit-QED system [6, 7]. An important difference is that the NV center qubit can be measured optically instead of being measured dispersively through the cavity. It means the high-loss cavity used for the fast measurement in the circuit-QED experiments [6, 7] is no longer necessary. We also note that our scheme offers orders of magnitude stronger coupling strength than recently proposed nanomechanical resonators-based quantum bus [18, 19] for the NV centers as well as schemes that use the flux qubits [15, 20], which enables us to perform fast quantum gate operations. Moreover, recent experimental demonstration of the coupling between an ensemble of NV centers and the superconducting resonator [13] along with experiments realizing the superconducting resonator with the Josephson junction inserted in the central conductor [17, 21] indicate a feasibility of our scheme.

*Resonator.*— We start with a general description of the resonator. We insert  $N$  junctions into the central conductor of the superconducting coplanar waveguide, essentially breaking the resonator into  $N + 1$  subres-

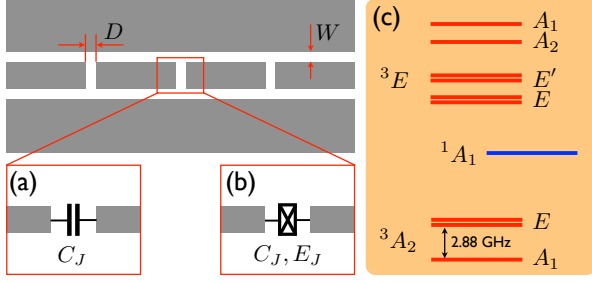


FIG. 1: (color online) A schematic diagram of the coplanar waveguide resonator with (a) capacitive or (b) Josephson junctions. (c) The energy-level structure of the NV center in diamond.

onators. The adjacent subresonators are coupled via the junctions. The central conductor of the  $r$ th subresonator occupies the space  $x_{2r} \leq x \leq x_{2r+1}$  with length  $\ell_r \equiv x_{2r+1} - x_{2r}$  ( $r = 0, 1, 2, \dots, N$ ) and has capacitance  $C_0$  per unit length. The junctions separating the adjacent subresonators have gap  $D_r = x_{2r} - x_{2r-1}$  ( $r = 1, \dots, N$ ) and junction capacitance  $C_J$ . We will assume that  $\ell_r \sim 10$  nm and  $D_r \approx 1$ –10 nm.

The Lagrangian  $\mathcal{L}_R$  governing the dynamics of the resonator has two parts:  $\mathcal{L}_R = \mathcal{L}_0 + \mathcal{L}_J$ . The first part  $\mathcal{L}_0$  describes the decoupled subresonators and is given by [8]

$$\mathcal{L}_0 = \frac{1}{2} C_0 \sum_{r=0}^N \int_{x_{2r}}^{x_{2r+1}} dx [(\partial_t \phi)^2 - v^2 (\partial_x \phi)^2] \quad (1)$$

where  $v \approx 10^8$  m/s is the propagating velocity of the electromagnetic wave in the coplanar waveguide. Physically, the field  $\phi(x, t)$  is proportional to the magnetic flux and related to the local electric potential  $V(x, t)$  by  $\partial_t \phi(x, t) = V(x, t)$ . The second part  $\mathcal{L}_J$  comes from the coupling between adjacent subresonators:

$$\mathcal{L}_J = \frac{1}{2} C_J \sum_{r=1}^N [(\partial_t \varphi_r)^2 - \omega_p^2 (\varphi_r)^2] \quad (2)$$

Here the variable  $\varphi_r(t)$  is the magnetic flux within the junction and related to the electric potential difference  $V_r(t)$  across the junction by  $\partial_t \varphi_r(t) = V_r(t)$ . The first term in Eq. (2) is thus responsible for the electric energy stored in the junction and the second, the magnetic energy.  $\omega_p = \sqrt{2E_C E_J}/\hbar \approx 2\pi \times 10$  GHz is the Josephson plasma frequency, where  $E_C \equiv (2e)^2/2C_J$  and  $E_J$  are the charging and Josephson coupling energy of the junction, respectively. The model (2) for Josephson junctions is valid only in the range  $k_B T \ll \hbar \omega_p \ll E_J$  whereas one can put  $\omega_p = 0$  in (2) for capacitive junctions. Note that for large  $D_r$  the junction capacity  $C_J$  (as well as the Josephson coupling  $E_J$ ) becomes very small compared with  $\ell_r C_0 \sim 1$  pF, and the coupling between adjacent

subresonators are negligible except for a small red-shift of order of  $C_J/\ell_r C_0$  [8]. For  $D_r \approx 1$ –10 nm,  $C_J/\ell_r C_0 \sim 1$  and the coupling is strong.

The field  $\phi(x, t)$  defined inside subresonators and  $\varphi_r(t)$  defined across the junctions are not independent. They are related to each other by the current conservation

$$\frac{C_J}{v^2 C_0} (\partial_t^2 + \omega_p^2) \varphi_r(t) = \partial_x \phi(x_{2r}, t) = \partial_x \phi(x_{2r-1}, t) \quad (3)$$

Following Ref. [16], we expand the field  $\phi$  and  $\varphi_r$  in the normal modes as

$$\phi(x, t) = \sum_{m=0}^{\infty} \phi_m(t) \psi_m(x), \quad \varphi_r(t) = \sum_{m=0}^{\infty} \phi_m(t) \Delta_{r,m}. \quad (4)$$

The eigenfunctions  $\psi_m(x)$  should satisfy the time-independent Schrödinger equation

$$(\partial_x^2 + k_m^2) \psi_m(x) = 0 \quad (x_{2r} \leq x \leq x_{2r+1}) \quad (5)$$

for some wave numbers  $k_m$ . Let  $\omega_m \equiv v k_m$ . The current conservation relation (3) now reads

$$\frac{C_J}{v^2 C_0} (\omega_p^2 - \omega_m^2) \Delta_{r,m} = \partial_x \psi_m(x_{2r}) = \partial_x \psi_m(x_{2r-1}) \quad (6)$$

Without loss of generality, we choose the normalization

$$\sum_{r=0}^N \int_{x_{2r}}^{x_{2r+1}} dx \psi_m \psi_n + C_J \sum_{r=1}^N \Delta_{r,m} \Delta_{r,n} = C_{\Sigma} \delta_{mn}, \quad (7)$$

where  $C_{\Sigma} = \sum_{r=0}^N \ell_r C_0 + N C_J$  is the total capacitance of the resonator. Putting the normal mode expansion (4) into Eqs. (1) and (2) and imposing the conditions (5), (6), and (7), one can rewrite  $\mathcal{L}_R$  into the simple form

$$\mathcal{L}_R = \frac{1}{2} C_{\Sigma} \sum_m [(\partial_t \phi_m)^2 - \omega_m^2 \phi_m^2] \quad (8)$$

By introducing a momentum  $\theta_m = C_{\Sigma} \partial_t \phi_m$  conjugate to  $\phi_m$ , we write the Hamiltonian of the resonator as

$$H_R = \frac{1}{2} \sum_m \left( \frac{\theta_m^2}{C_{\Sigma}} + C_{\Sigma} \omega_m^2 \phi_m^2 \right) \quad (9)$$

We then quantize it by the canonical commutation relation  $[\phi_m, \theta_m] = i\hbar$ . It is customary to introduce the annihilation and creation operators of the normal modes by the relations

$$\phi_m = \sqrt{\frac{\hbar}{2\omega_m C_{\Sigma}}} (a_m^\dagger + a_m), \quad (10a)$$

$$\theta_m = i\sqrt{\frac{\hbar\omega_m C_{\Sigma}}{2}} (a_m^\dagger - a_m), \quad (10b)$$

in terms of which the resonator Hamiltonian reads

$$H_R = \sum_{m=0}^{\infty} \hbar \omega_m a_m^\dagger a_m \quad (11)$$

Due to the nonlinearity induced by the junctions,  $\omega_m$  determined by (6) is not an integer multiple of the  $\omega_0$ .

*Coupling to NV centers.*— We now describe the diamond NV centers and their coupling to the resonator. The ground-state triplet (spin 1) of the NV center has a level splitting  $\epsilon/2\pi = 2.88$  GHz due to the spin-spin interaction [22, 23], and thus is described by the Hamiltonian

$$H_{NV} = \hbar\epsilon \sum_{r=1}^N S_{r,z}^2 \quad (12)$$

where  $S_{r,z}$  is the spin  $z$  component of the NV center at the  $r$ th junction.

The ground-state spin triplet is coupled magnetically to the resonator, which is governed by the coupling Hamiltonian

$$H_g = \sum_{r=1}^N g_e \mu_B S_{r,z} B_r \quad (13)$$

where  $B_r \equiv \varphi_r/D_r W$  ( $W$  is the distance between the central conductor and the ground plates) is the local magnetic field at the junction  $r$ ,  $g_e \approx -2$  is the electron  $g$ -factor, and  $\mu_B$  is the Bohr magneton.

Using the annihilation and creation operator defined in Eq. (10), the coupling Hamiltonian is written as

$$H_g = \sum_{m,r} g_{m,r} (a_m^\dagger + a_m) S_{r,z} \quad (14)$$

Here we have defined the coupling constant

$$g_{m,r} = g_e \mu_B B_{m,r} \quad (15)$$

where  $B_{m,r}$  is the root-mean square value of the local magnetic field in the mode  $m$  at the junction  $r$ . By adjusting one of the mode frequencies at resonance with the level splitting of the NV centers ( $\omega_m \approx \epsilon$ ), one can selectively couple the mode  $m$  to the NV centers.

Putting all together, the total Hamiltonian is given by the sum  $H = H_R + H_{NV} + H_g$ . The description above is completely general (in principle) for any number of junctions and NV centers. Equations (5) and (6) determines the possible spectrum  $\omega_m$  ( $m = 0, 1, 2, \dots$ ) of the coupled subresonators whereas the normalization (7) determines the magnitude of the magnetic field at the junctions. Below we demonstrate the cases with single NV center and two NV centers.

*Single NV center.*— We assume that the junction is located at  $x = 0$ , and the two subresonators are identical with length  $\ell$ . The wave function  $\psi_m(x)$  of the normal mode  $m$  has the form[16]

$$\psi_m(x) = A_m \begin{cases} +\cos[k_m(x+\ell)] & (x \leq 0) \\ -\cos[k_m(x-\ell)] & (x > 0) \end{cases} \quad (16)$$

The wave number  $k_m$  and the frequency  $\omega_m = vk_m$  are determined by the current conservation relation (6),

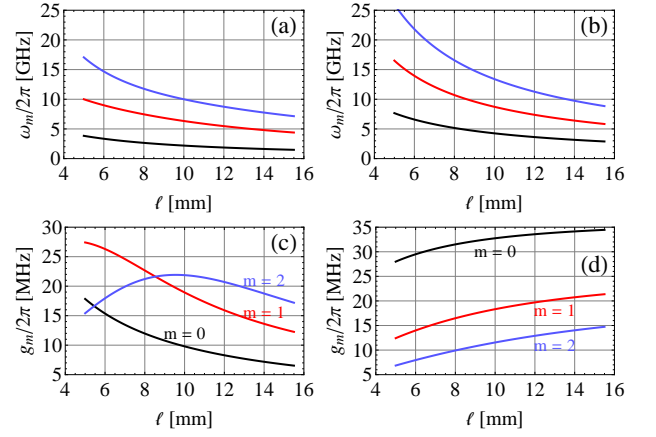


FIG. 2: (color online) (a,b) Resonance frequencies  $\omega_m$  of the modes  $m = 0, 1, 2$  (from bottom to top) as a function of the overall length  $\ell$  of the resonator for  $\omega_p/2\pi = 10$  GHz (a) and  $\omega_p = 0$  (b). (c,d) The coupling strength  $g_m$  to the modes  $m = 0, 1, 2$  for  $\omega_p/2\pi = 10$  GHz (c) and  $\omega_p = 0$  (d).  $C_J = 0.15$  pF,  $C_0 = 0.16$  pF/mm,  $D = 5$  nm, and  $W = 10$   $\mu$ m.

which in this case reduces to

$$\frac{2C_J}{v^2 C_0} (\omega_p^2 - \omega_m^2) \cot[k_m \ell] = k_m, \quad (17)$$

and the constant  $A_m$  (whose explicit expression is not given) is determined by the normalization condition (7). The coupling strength  $g_m$  in Eq. (15) takes the form

$$g_m = \frac{4\mu_B A_m \cos(k_m \ell)}{DW} \sqrt{\frac{\hbar}{2\omega_m C_\Sigma}} \quad (18)$$

Figure 2 shows  $\omega_m$  and  $g_m$  as a function of the length  $\ell$  for a few lowest modes. It demonstrates that the coupling strength is in the strong coupling regime ( $g_0/2\pi \sim 10$  MHz) when the resonator is close to resonance to the diamond NV center ( $\omega_m \approx \epsilon$ ). Notice that a capacitive junction gives coupling as strong as a Josephson junction. A submicron size of the gap junction has been made on coplanar waveguide [24].

*Double NV center.*— Let us now consider the case with two NV centers. For simplicity, we assume symmetrically located NV centers with subresonators of lengths  $\ell, \ell', \ell$  in this order ( $x_5 = -x_0 = \ell + \ell'/2$ ,  $x_4 = -x_2 = \ell'/2$ , and  $D = x_2 - x_1 = x_4 - x_3 \ll \ell, \ell'$ ). Because of the symmetry, a normal-mode wavefunction  $\psi_m$  has either even or odd parity. Here, we only consider the lowest mode, i.e., the  $\lambda/2$  mode. The eigenfunction takes the form of

$$\psi(x) = A \begin{cases} \frac{\sin(k\ell)}{\cos(k\ell'/2)} \sin(kx) & (0 \leq x \leq \ell'/2) \\ \cos k(x - \ell - \ell'/2) & (\ell'/2 \leq x \leq \ell) \end{cases} \quad (19)$$

The mode frequency  $\omega = vk$  is determined by the current conservation (6)

$$vC_0\omega = C_J (\omega_p^2 - \omega^2) [\cot(k\ell) - \tan(k\ell'/2)] \quad (20)$$

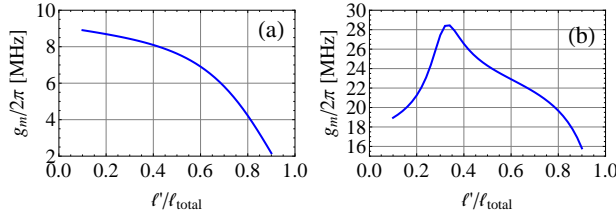


FIG. 3: The coupling strength (a) for  $\omega_p/2\pi = 10$  GHz and  $\ell_{\text{total}} = 18$  mm and (b) for  $\omega_p/2\pi = 0$  and  $\ell_{\text{total}} = 25$  mm.  $C_J = 0.15$  pF,  $C_0 = 0.16$  pF/mm,  $D = 5$  nm, and  $W = 10$   $\mu$ m.

while the constants  $A$  (not given here) is determined from the normalization condition (7). The coupling strength at the junctions is found to be

$$g = \frac{2\mu_B A}{WD} \sqrt{\frac{\hbar}{2C_{\Sigma}\omega}} [\cos(k\ell) - \sin(k\ell) \tan(k\ell'/2)] \quad (21)$$

The coupling strength  $g$  as a function of  $\ell'$  is shown in Fig. (3).

Let us now estimate the coupling strength  $J$  between two NV centers through a virtual excitation of the photon in a dispersive regime. For the resonators coupled through Josephson junctions, we have the frequency,  $\omega/2\pi = 2.4$  GHz, and the NV center-resonator coupling,  $g/2\pi \sim 8$  MHz, for  $\ell_{\text{total}} \equiv 2\ell + \ell' = 18$  mm and  $\ell' \sim 0.4$  mm. Assuming a detuning of  $\Delta/2\pi = 200$  MHz between  $\epsilon$  and  $\omega$ , we obtain the coupling strength between the NV centers  $J = g^2/\Delta \sim 2\pi \times 320$  kHz. For  $Q = 10^5$ , the cavity decay rate is  $\kappa = \omega/Q \sim 2\pi \times 20$  kHz which is lower than the coupling strength  $J$ . Even for  $J \sim \kappa$ , two-qubit quantum gates can be performed with a high-fidelity as demonstrated in Ref. [7]. Thus, given the spin coherence time of 1 – 10 ms [25], a high fidelity quantum gate is realizable using our scheme.

*Discussion* – Above we have focused on how to strongly couple diamond NV centers to the resonator. The strong coupling opens another interesting possibility to couple NV centers to other types of superconducting qubits [6, 8–10, 16, 17, 26] through the resonator. This way one can take the best of features that each qubit provides. It will ultimately provide a novel architecture for quantum information processors integrating diamond NV centers and superconducting qubits into the circuit QED system.

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